Might want to consider more problems made from SF 2048 relating k to Y.

**Example**

If the Young’s modulus of steel is Y = 2×1011 Pa. Since it is mostly iron, assume steel has the density and molar mass of iron. Then, according to Wikipedia, we’d have ρ = 7874 kg/m3, and mmol = 55.85 g/mol. So what is the spring constant of the interatomic bonds?

From our equation above,



and from further up,



and so,



That’s a pretty small force on our size scale (10lbs), but consider that this force is being exerted on molecules. Adjusting for size – it’s a huge force. And note that this matches the order of magnitude of atomic force F = ke2/r = 101010-38/(10-9)2 = 10-8, and F = kx ~ 10×10-9 = 10-8.

1. A tow truck is pulling a car out of a ditch by means of a steel cable (*Y* = 2.0 x 1011 N/m2) that

is 10 m long and has a radius of 0.5 cm. When the car just begins to move, the tension in the cable is 1500 N. How much has the cable stretched? D

The stretch of the cable can be determined from Young’s equation:



2. An aluminum baseball bat has a length of 0.75 m at a temperature of 20.0°C. When the temperature of the bat is raised, the bat lengthens by 0.4mm. Determine the final temperature of the bat. Note that αAl = 23.1×10-6 K-1. D

Using the thermal expansion equation:



So the final temperature is T = T0 + ΔT = 43.1˚C

**Example:**

How much does a 25m long steel cable (cross section d = 2cm) stretch when holding an elevator with 5 people in it. Assume the mass of the elevator and people is 1000kg.

First we’ll look up Y for steel. It is 2×1011Pa. So then according to the formula, we have,



**Example:**

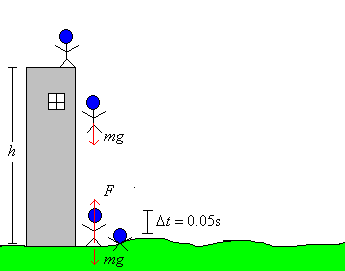
How much does a 25m long steel cable (cross section d = 2cm) stretch when we increase its temperature from 25˚C to 85˚C.

First we’ll look up α for steel. It is α = 1.2×10-5 ˚C-1. So then according to the formula, we have,



**Example:**

If the elastic modulus of your shin bone is 1.4×1010Pa, and has a cross section diameter of d = 2cm, from what height building can you jump without breaking your leg, assuming the collision between the ground and you lasts 0.05s? Suppose you weigh 70kg.



Working backwards, your leg will break if the strain ΔL/L on it is approximately 1%, which would correspond to a force on it of:



Now since you have two legs, we will assume that the ground can exert a maximum force of F = 88 000 N on you just before your legs break. Now if the ground exerts this force on you for Δt = 0.05s to bring you to rest, then you must have had a velocity, just before hitting the ground, of,

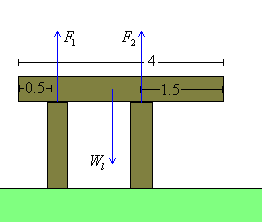


Now the question is, from what height must you fall in order to have a velocity of 62m/s when you hit the ground? We’ll use the WE equation,



**Example:**

Consider the temple again. The cross-beam (lintel) has a mass m = 5000 kg. And suppose that the posts have a cross section A = 0.03m2. What is the percent deformation of the two posts assuming Y = 3×1010 Pa? For now we’ll assume the posts themselves are massless.



First we label the forces. F1 and F2 are obvious. Wℓ is the weight of the lintel acting at its center of mass (the middle of the lintel assuming its uniform). We can use N2L and the torque equation to get F1, and F2. Repeating our previous work we get, F1 = 12 250N, and F2 = 36 750 N. Thus, the % deformation of the left post is:



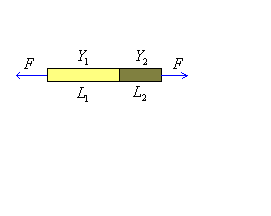
So the left post will shrink by this fraction. And of the second post,



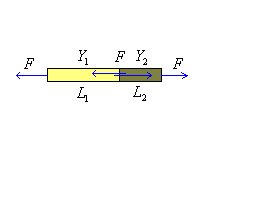
and so the right post will shrink by this amount.

**Example:**

Suppose you have two materials stuck together, one with modulus Y1 = 2×1011, and another with Y2 = 1×1011. And further, suppose L1 = 2.2m, and L2 = 1.2m. Lastly, assume the cross section area is A = 5cm2. If you pull on both ends with a force F = 950N, what will be the stretch of the composite bar?



To proceed we have to determine the forces on each piece separately. So consider that material 1 must exert a force F on material 2 (since material 2 isn’t going anywhere), and likewise, material 2 must exert a force F on material 1. So we have the following picture,



The stretch of material 1 will be, according to Young’s equation.



The stretch of material 2 will be given by:

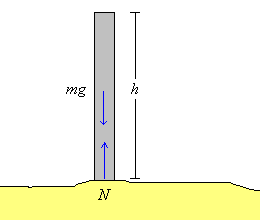


So the total stretch will be:



**Example:**

Consider building a concrete tower with length ℓ = 50m. If Y = 8×1010 Pa, and ρ = 2400 kg/m3, what is its new height when it is stood up?



The compression of the tower will be given by:



The force at the base of the object is N = mg, and the force at the top is 0. Now remember that gravity has no effect on the *overall* compression of the object. So the stretch is given by:



Filling in the values we get,



and so the new height will be:



**Example:**

Let’s approximate a tennis ball as a cylinder (m = 200g, A = 2×10-3m2). Suppose it’s coming towards you with a speed of 40m/s, and you hit it with your tennis racket so that it flies backward with a speed of 50m/s. If the collision lasts 0.02s, and Y for the tennis ball is Y = 1×104 Pa, what is the % deformation of the ball, ΔL/L?

From the impulse-momentum equation, the force acting on the ball is:



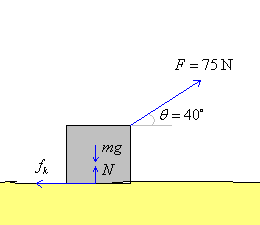
Therefore the percent deformation will be:



**Example:**

Suppose you drag a cubical (ℓ = 1.2m) 10 kg box along the ground with a force F = 75N at an angle of 40˚ w/r to the horizontal. If its shear modulus is S = 2×102 Pa, what will be its shear? What will be its sheer if there is a coefficient of friction μk = 0.2? What will be its compression in the vertical direction, if its Young’s modulus is Y = 6×102 Pa? Finally, what is the block’s acceleration?

First we’ll draw all the forces acting on our box.



Assuming it isn’t accelerating in the y-direction, we have,



The vertical elongation of the box can be determine then to be:



Note that we have to isolate the vertical components of the forces acting on our box. Also, we need to take into account the sign of the F. It get’s a negative sign because its going in the direction opposite to that of a compression force.

Now for the shear. First assume no friction, then we have,



If friction is present, then the friction force is



in which case the shear would be,



If friction is present, then the acceleration of the block would be:



**Example:**

Estimate how much will a steel ball will expand if it is shot into outer space?

The bulk modulus for steel is something around its elastic modulus which is around 20×1010 Pa. If we shoot it into outer space, then the pressure goes from 1atm (Patm = 1.01×105 Pa, i.e., the atmospheric pressure at sea level) to 0atm in outer space. So ΔP = -Patm. Therefore the fractional change in volume of the steel ball will be:



- not very much clearly.

**Example**

When water freezes into ice, it would, left to its own, expand by about 9%. Suppose you didn’t use anti-freeze in your coolant one winter, and the water froze in the pipes. What pressure would the pipes have to be able to withstand, in order to keep the ice from expanding?

To answer this question, we would rephrase it this way. What pressure would the pipes have to exert to compress ice by 9% (back to its original volume)? The answer is given by Young’s equation,



So they’d have to withstand quite a large pressure. They probably wouldn’t be able to, and would therefore break. This is why you want the anti-freeze, so the ice expansion deal doesn’t happen.

**Example: bulk modulus for a gas at constant temperature**

Let’s consider an ideal gas, maintained at a constant temperture. What is B for it?

Well, according to the ideal gas equation we have PV=nRT. Now take the derivative of P w/r to V and we get,



Now rearrange this equation…



and we can identify nRT/V as the bulk modulus, B. So we have,



**Example:**

How much does a 25m long steel cable (cross section d = 2cm) stretch when we increase its temperature from 25˚C to 85˚C.

First we’ll look up α for steel. It is α = 1.2×10-5 ˚C-1. So then according to the formula, we have,



**Example:**

Estimate how (percent) much will a steel ball contract expand if it is shot into outer space? Assume that its temperature on Earth is 25˚.

First, β = 3α = 3.6×10-5 ˚C-1 for steel. The writing out the equation…



So it will contract by 1.1%. Of course, due to the pressure drop, it would also expand, as we saw in the last lecture. The net effect would be the sum of both of these. The compression is the larger effect though.